Torsten Ekedahl: some recollections

Luc Illusie

At the end of the summer of 1980 I received an astonishing letter. Its author, a certain Torsten Ekedahl, wrote to me: "I have obtained some results on the slope spectral sequence, some of which are perhaps unknown to you. (…)"[1] At that time I was busy together with Michel Raynaud in preparation of an article on the de Rham–Witt complex.[2] Some of the results presented by Ekedhal were known to me but were proved in a shorter and more elegant way. Others, which I had expected to prove but which had resisted all my attempts, were proved with the same ease. A little later, in a subsequent letter, Ekedahl explained to me the solution of a problem whose formulation had even appeared intractable to me: duality in the theory of de Rham-Witt complexes. Once again, the method was very natural and the proof that he sketched very convincing. That was the starting point of an intense correspondence. It was not until later, when Ekedahl went to Orsay in order to finish the preparation of his thesis, that I became privy to how this young student – he was 25 at the time – had taken an interest in this sophisticated theory, which at the time was quite mysterious (and, I fear, remains so today). In July 1978, he was on vacation in Brittany. He had heard of a meeting which was taking place in Rennes, the Journées de géométrie algébrique.[3] Out of curiosity, he went into the lecture hall and listened to the talk I was giving on the de Rham-Witt complex and its relations to crystalline cohomology. Thrilled, he decided to work on the subject. But he made no contact with me so I had no inkling of that.

The year of his stay at Orsay, 1981–82, was one of the most rewarding in my career. I helped him in writing up his thesis and asked him questions. We would see each other practically every day. He resolved all my questions one by one, constantly introducing new ideas. In principle I was his advisor but I often had the impression that I was actually his student. The Künneth formulas in de Rham–Witt theory seemed even more inaccessible than those on duality. Child's play to him. In the Autumn of 1982 there was a conference on algebraic geometry in Japan.[4] I gave a two-hour survey on his work. And thereafter there was the memorable defence of his thesis in Gothenburg in 1983. I played the role of the opponent. In order to trick him, I asked him questions about signs and commutativity of diagrams. Wasted effort. That evening at his home with his family we celebrated the occasion in style, downing a raw, homemade aquavit.

Afterwards we ran into each other frequently, especially in the 80s and 90s. He quickly turned to other subjects, such as surfaces, foliations and moduli spaces, each of which received the spark of his genius. The qualities that first come to my mind were his gentleness, his modesty and his generosity, and his sharp, Bourbaki-like way of tackling problems, coupled with his ability to think in an unconventional fashion, 'penser à côté' (to go against the grain) in the words of Hadamard. How many times were we not on the phone! I can still hear his voice when he picked up the receiver: 'Torsten'. And then began, in French, a language he mastered to perfection, a rich and stimulating conversation.

Adieu, Torsten

Paris, 27 November 2011

Translated from the French by Ulf Persson. The article was initially solicited by the Swedish Mathematical Society, delivered with commendable promptness and published in a Swedish translation by the editor Per-Anders Ivert in the first issue of its new newsletter – SMS Bulletinen – in December 2011. The original will be published in the French journal Gazette des Mathématiciens.

Torsten Ekedahl

Jan-Erik Roos

Torsten Ekedahl is dead. He collapsed at the Department of Mathematics of Stockholm University on the morning of 23 November 2011. Attempts at resuscitation by colleagues and paramedics were to no avail. The cause was in most likelihood a massive heart-attack. He was active until the very end. Just a few hours before, he had been logged in on the site "mathoverflow" to which he was a much appreciated contributor. He was 56 years old. It is a very big loss to Swedish mathematics, and many of us have not only lost a good friend but also a passionate and exceedingly knowledgeable discussion partner.

My first real contact with Torsten Ekedahl occurred when I was President of the Swedish Mathematical Society (1980–82). An anonymous benefactor had for many years donated money to the prizes given out to the winners in the Swedish High-school competition in mathematics. I thought (inspired by the AMS) that it would also be a good thing to give stipends to young, promising mathematicians who had just written their dissertation, thus enabling them for the next few years to continue

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[1] In English in the original. Translator's remark

[2] L. Illusie et M. Raynaud, *Les suites spectrales associées au complexe de de Rham–Witt*, Publ. math. I.H.E.S. 57 (1983), 71–219.

[3] *Journées de Géométrie Algebrique de Rennes, I, II, III*, Eds. P. Berthelot, L. Breen, Astérisque 63, 64, 65 SMF, 1979.

[4] *Algebraic Geometry,* Proceedings Tokyo/Kyoto 1982, Eds. M. Raynaud, T. Shioda, Lecture Notes in Mathematics 1016, Springer-Verlag.

their research without any material worries. At that time this was not so easy to arrange in Sweden. The donator liked the idea and provided the necessary means. It was decided that the first stipend would be awarded in 82/83 after I had been succeeded by Lars-Inge Hedberg as president. Professors in mathematics and adjacent subjects in Sweden were invited in the Autumn of 1982 to suggest candidates, and a committee consisting of myself, Björn Dahlberg and Hedberg was set up as a jury to make a final selection out of the 13 candidates submitted. Torsten Ekedahl was submitted from Gothenburg. It was natural for me to contact Heinz Jacobinski in Gothenburg, whom I knew well from our time in Lund. His reaction was astounding to me. He compared Ekedahl with Hörmander.

The referee reports we requested of the candidates were all positive but in the end it became clear that Ekedahl was in a class all by himself. Torsten received the stipend in January 1983 and a few years later the anonymous donator would be replaced by the Wallenberg Foundation after which the stipend would be renamed. Torsten defended his thesis in Gothenburg on 28 May 1983. His advisor was Juliusz Brzezinski who had played an important supporting role in Torsten's mathematical development, although he was more specialised in his interests. How could Torsten make such an important contribution to a subject so very far away from results that had been obtained earlier in Swedish mathematics? The answer, of course, is to be found in his inherent ability as a mathematician but also in his passionate interest and his ultimate goal to understand all of mathematics.

Torsten was born in Lund in 1955. In the Autumn of 1974 he shared first prize in the mathematical high-school competition referred to above and graduated from high-school the following Spring in Helsingborg. He took his basic university degree (fil.kand) in Gothenburg in 1977. But before he got a stipend for graduate study he had hit upon unconventional ways of furthering his studies and research. The following story is typical (he told it to me himself). When Torsten was on an inter-rail vacation in Brittany he learned coincidentally that an international conference on algebraic geometry was taking place in Rennes (3-7 July 1978). Torsten went there by train and attended the lectures on the first day, no doubt already making comments in the style of which he would later excel, and returned in the evening to his sleeping bag in the train station. This was repeated four times until the conference came to an end. He made valuable contacts, notably with Nicholas Katz from Princeton, but above all he became inspired by the lectures given by Luc Illusie from Paris. Later Torsten was given a doctoral stipend which enabled him to study algebraic topology at Århus (1980/81) while concomitantly developing the threads he had picked up in Rennes. On the strength of the results he achieved in the Summer and Autumn of 1980 he was invited to IHES (1981/82). His crucial move was to seek out Illusie, who later wrote: "He asked me to give him some guidance. I did my best, but quickly the opposite occurred: he was the one who guided me!"

His dissertation dealt with cohomology of algebraic varieties defined over a field of finite characteristics. At the time there were at least three different cohomological theories: the Witt-vector cohomology of Serre, the so-called crystalline cohomology due to Grothendieck and Berthelot and, finally, the Hodge groups, which each, in its way, gave important information about the structure of the varieties. Spencer Bloch and others had introduced a so-called Rham–Witt complex which was intended to connect those various theories and led to a unified theory (de Rham–Witt (hyper)cohomology). This had been studied by several distinguished mathematicians and yet many problems remained, such as duality and multiplicative structure. Ekedahl solved these problems in a natural and elegant way, which had occurred to neither Deligne nor Illusie and which harnessed all the modern algebraic tools available. An excellent survey of all of this is to be found in an article by Illusie in SLN 1016. But, in spite of this predilection for abstraction, Torsten was not a stranger to very concrete applications, which are to be found in his thesis. When he applied for a professorship at Stockholm in 1988, Atiyah was very impressed by his ability to combine abstract theory with concrete results.

After his dissertation Torsten explored many other mathematical avenues with a more classical flavour. One may as an example mention that he showed in an elegant article that the results of Deligne–Griffiths–Morgan–Sullivan on rational homotopy theory of complex projective manifolds were the best possible, in the sense that the Massey-products modulo p could be non-zero for those manifolds. He also showed in a longer work, subsequently to be published in the IHES-series, that many of the standard results on surfaces in characteristic 0 could be extended to finite characteristics, although Kodaira's vanishing theorem did not apply. Miyaoka judged it as "[…] a fundamental contribution to the theory of algebraic surfaces in positive characteristics".

Furthermore, he proved a generalization of Hilbert's irreducibility theorem, which has often been cited, and he had tentative ideas about extending rational homotopy theory to a theory over the integers. He often spent time at IHES but did not have a permanent position of any kind. In 1984, due to a successful evaluation of mathematics, a further research position, a so-called docent position, was created at Stockholm University. Many worthy candidates applied and an expert committee headed by Deligne and Yves Meyer awarded Torsten the position. In that way he became attached to Stockholm University and when a new professorship was created there by the Government in 1988 it was his for the taking. (Incidentally, three Fields Medallists served on the expert committee: Faltings and Hörmander in addition to the aforementioned Atiyah.)

After Torsten became a professor he developed his research interests in all kinds of directions and it is impossible to describe everything he did.

He wrote monographs and collaborated with many mathematicians. He was very happy about a recent collaboration with the combinatorialist Anders Björner applying "étale intersection cohomology" to derive unexpected results about the Bruhat order (*Annals of*

Torsten Ekedahl and Kathryn Hess acting in the play "Fermat's riddle", November 2000

Math. 170, 2009). But he also had joint publications with Gerard van der Geer, the brothers Boris and Michael Shapiro, Nick Shepherd-Barron, Dan Laksov, Trygve Johnsen, Dag Einar Sommervoll, Pelle Salomonsson and not to forget Jean-Pierre Serre. The last joint publication has an interesting story. When Torsten in 1989 attended a conference on the Dutch Frisian island of Texel he got involved in a "competition" with Serre to find examples of the Jacobian of a curve essentially decomposing as a product of elliptic curves. (In classical language, when can certain abelian integrals on a complex curve be written in terms of elliptic integrals?) It is unknown if you can find examples for arbitrarily high genera. It has never been clear to me who "won" but, according to Torsten, Serre thought after a while that they would stop competing (it had by then been brought up to genus 1297). (See the joint note in *Comptes Rendus* 317 (1993), 509–513, as well as comments in the collected works of Serre volume 4.) But Torsten worked with many others, e.g. Carel Faber (they arranged a year on moduli spaces at Mittag-Leffler), Roy Skjelnes, Sergei Merkulov, Sandra di Rocco, Wojciech Chacholski, Richard Bøgvad, Ralf Fröberg, Leif Johansson, Lennart Börjeson, Tomas Ericsson and me. He has also been a very active advisor of many graduate students, out of whom Alexander Berglund can be mentioned as the recipient of several awards.

Torsten was elected a member of the mathematical section of the Swedish Royal Academy of Sciences in 1990. His wide culture was invaluable and his academy lectures to a general educated public on such varied subjects as the Riemann Hypothesis and the results of Perelman on 3-manifolds testifies that he took the obligation to reach the general public seriously. The last lecture will be available until 2015 on http://urplay.se/162252 and it is, incidentally, the best of its kind I have heard and seen of him. Torsten managed by computer animation to show the flow of Perelman's proof without any unnecessary formalism. This really impressed the audience including the attending non-mathematicians.

In addition, Torsten got the Göran Gustafsson Prize in mathematics in 1994, he was a member of the board of the Mittag-Leffler Institute, he worked for the National Swedish Science Foundation and he was also dean for a section of the science faculty at the university.

Torsten was a natural talent. Whatever he touched he always contributed a new thought or a different perspective. This applied not only to mathematics, theoretical physics and computer science but also to other fields of human endeavour. His encyclopaedic erudition and sound judgement made him a much sought-after general lecturer, a member of various expert committees and a referee. One may forgive him if his generosity paired with kindness and a general inability to say no saddled him with too many refereeing assignments that were invariably delayed. But his opinions on important questions were very well thought through and were enormously appreciated.

When a mathematician dies it is common to say he will survive through his publications. This is also true for Torsten but in his case with the addendum that many of his ideas remain half-complete. He had many pending projects, on his own as well as with collaborators, which he would have had the capacity to successfully bring to fruition. He also had inspiring ideas about future research projects which he had not had time to make more precise. In connection to being awarded the Gustafsson Prize (referred to above), one of the referees wrote about one of his "visionary ideas": "I would almost apply for a second life as a mathematician in order to be able to go into this direction myself."

Sweden has lost one of its foremost mathematicians and we miss him very much.

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