

The Wandering Must Continue

Pure Mathematicians as Rangers of the North

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I. THE SEMINAR LIST

I have a test. When I travel to a country I have never visited and want to understand its scientific and technological capacity — not its GDP, not its export statistics, not its startup culture — I go to the best university in that country. I walk into the mathematics department. I find the seminar schedule, usually posted on a board near the entrance or taped to a wall on the first floor. And I read the titles.

That is all I need.

If the titles read something like “Distribution of Hodge loci and atypical special points” or “Resolution of singularities in positive characteristic” or “Unboundedness of Mordell–Weil rank over global fields,” I know that this country maintains deep scientific infrastructure. Not because anyone will ever apply these results to build a bridge or cure a disease — almost certainly no one will — but because the presence of people doing this kind of work is a reliable signal that the broader ecosystem is healthy. Good physicists were trained here. Good engineers passed through the same corridors. Good AI researchers working on alignment took the same foundational courses.

Applied titles tell me other things, also worth knowing: how deeply the country’s scientific expertise reaches, what level of analysis its ministries bring to strategy, where the ceiling lies on nonlinear physical modeling built on rigorous numerics. Between pure and applied mathematics runs a supply line, not a wall, and traffic moves in both directions. But it is the pure titles that make the test work, for a simple reason: nobody has an instrumental motive to fake them. No ministry ever demanded a seminar on Hodge loci. No investor was ever impressed by one. If it is on the board, it is there because the culture that produces such things is alive in the building. That is what makes the signal honest.

This is an observation about national infrastructure. The seminar list is a barometer, and like all barometers, it measures something other than what it displays. It displays abstruse mathematics. It measures the depth of a civilization’s capacity to reason about the structures that underlie its own technology.

II. RANGERS OF THE NORTH

Tolkien understood something about invisible protection.

In *The Lord of the Rings* [1], the Shire, that green, peaceful corner of Middle-earth, is kept safe not by its own inhabitants but by the Rangers: grim, weathered men who patrol its borders in silence, fighting threats that hobbits have never seen and do not believe exist. The hobbits regard the Rangers with suspicion and faint contempt. “He is one of the wandering folk — Rangers we call them,” says the innkeeper at Bree, in a tone that suggests vagrancy more than vigilance.

There is a verse about Aragorn that Tolkien places early in the story, a riddle that doubles as a thesis:

*All that is gold does not glitter,
Not all those who wander are lost;*

The Rangers do not glitter; they produce nothing visible. They hold no office, collect no revenue, build no monuments. They wander the wild lands in the dark, and the hobbits, safe behind borders they have never had to defend, brew their ale and smoke their pipes and wonder why anyone would choose such a life.

I think about the Rangers more often than I should, because they describe, with uncomfortable precision, the societal position of pure mathematicians.

III. WHAT MATHEMATICIANS ACTUALLY DO

The public image of a mathematician is a person who solves very hard problems. This is not entirely wrong, but it is wrong in the way that matters. It is like saying that a physician is a person who writes prescriptions: technically true, and fundamentally misleading.

What mathematicians actually do, most of the time, is maintain a culture. They sustain a set of practices, habits of thought, standards of argument, and norms of honesty that have been built up over centuries and that cannot be written down in any manual. A graduate student does not simply learn how to prove new theorems. She learns how to be suspicious of her own reasoning. She learns that an argument that “feels right” is not right until every step has been verified. She learns to be grateful when someone finds an error in her work, because the alternative, an error that persists, is worse, and that stating a result precisely is as important as proving it.

But that discipline is only the preparation for a deeper transformation. At some point in her apprenticeship, a threshold is crossed: she stops experiencing mathematics as a game of rules played with symbols on paper and begins to see the structures themselves. She learns to reach for the right geometric principle or algebraic structure, so that a new theorem does not merely stand as an isolated fact, but illuminates the landscape around it. What once looked like formal manipulation becomes an act of perception, an encounter with objects that have their own stubborn necessity. She learns that the difference between a good mathematician and a mediocre one is not speed or

cleverness but a quality of attention and depth of awareness: a refusal to let anything pass that has not been fully understood, or to compromise true understanding for a mechanical answer.

These habits of mind and sight are not natural. They are not inevitable. They must be cultivated, by example, over years, in academic communities of people who share them. They cannot be acquired from a textbook any more than surgical skill can be acquired from an anatomy atlas.

And they are, I submit, among the most valuable cultural artifacts that human civilization has produced. Their value lies not in the theorems they generate, most of which are read by a few dozen people and forgotten, but in the habits of rigor, honesty, and precision that radiate outward from mathematics departments into every field they touch. You see their trace in the physicist who distrusts a convenient heuristic until the derivation is sound, the engineer who refuses to shave a structural safety margin, and the AI researcher who checks every edge case rather than assuming the pattern holds. These instincts were shaped, directly or indirectly, by contact with a culture that treats the difference between “almost certainly true” and “proved” as the difference between nothing and everything.

The patrol that mathematicians conduct, against imprecision, against the lure of the plausible-but-unverified, is invisible to everyone outside the community. Its work is quiet. Its victories make no headlines. It has no market value, no political constituency, no public advocate.

And it protects everything.

IV. WHAT CANNOT BE RECONSTITUTED

The danger is not that mathematicians will be replaced overnight; it is slower, quieter, and for that reason harder to resist.

A university facing budget pressure does not abolish its mathematics department. It freezes a position here, merges a program there, eliminates a one-year foundational course on algebraic geometry. The seminar list gets a little shorter. The graduate program admits one fewer student. A postdoctoral fellowship is converted to a teaching position. Each individual decision is rational. Each is, on its own, minor.

The result, compounded over a decade, is that this *culture* begins to thin. Not to vanish entirely, but to thin, the way a forest thins when its oldest trees are not replaced. You can still walk through it. You can still call it a forest. But its *capacity* to sustain the *ecosystem* it once supported has quietly diminished.

This process is already underway. We see it in quiet, localized attrition, and increasingly, in structural amputation. In September 2023, West Virginia University, a flagship state research institution, approved discontinuing its master’s and doctoral mathematics programs to close a

budget deficit [2]. The mechanism is not dramatic: no one is burning libraries. The mechanism is arithmetic: fewer positions, fewer students, fewer grants, fewer seminars. The barometer drops.

The critical fact about mathematical capacity is that it cannot be reconstituted on demand. You cannot mass-produce a mathematician through an accelerated three-month training program, the way you might scale a data center. The training takes ten to fifteen years and depends on a tradition of apprenticeship that stretches back centuries, because what is being transmitted is not a manual of procedures, but a way of seeing, kindled from one mind to another.

It took three thousand years to build this chain. It would take a generation to break it and perhaps a century to notice what was lost.

V. THE CERTIFICATE AND THE CANDLE

Here I must answer the obvious objection, because every technically literate reader has already raised it. We are entering an era where artificial intelligence does not merely verify mathematics, but autonomously discovers it. OpenAI has recently announced an AI-generated proof that the three-dimensional incompressible Navier–Stokes equations, with smooth external forcing, can develop a singularity in finite time, and has released a formalization in Lean [3]. Formal proof assistants check every step of an argument against the axioms, mechanically and without fatigue. A correctly checked formal proof establishes that its stated conclusion follows from the declared axioms and definitions, subject to the correctness of the proof-checking system. If AI systems produce the theorems and proof assistants certify them, the pipeline from conjecture to formally verified result appears complete. Why maintain, at enormous expense, a guild of artisanal mathematicians?

It is a good objection, and I concede everything in it except the conclusion.

A proof certificate establishes that a conclusion follows from the formal assumptions. By itself, it need not make the argument’s central ideas intelligible or show how they transfer to another problem. When an autonomous system produces a large, independently checked formal proof establishing a finite-time singularity under specified hypotheses, we can have every reason to trust the certificate while still lacking an illuminating account of the mechanism or its broader implications. The danger of our current trajectory is not that deep neural networks can generate mathematics and proof assistants can verify it. The danger is the seductive economic conclusion that administrators and those who fund science could draw: that because generation and verification can now be automated, human comprehension no longer needs to be subsidized. The machine may supply the verification, but human judgment must reclaim the meaning. Establishing a result and understanding why it holds are distinct achievements.

There is a deeper problem. Someone must assess whether the formal statement captures the intended question, even when a machine helped formulate it. A proof assistant will happily certify

a flawless proof of a subtly different theorem. Deciding what to prove, and judging that a formal statement means what we intend it to mean, that the tangible string of symbols captures the intangible thing we should care about, is not clerical work, but an act of human will: a reach toward conceptual elegance that is arguably the hardest part of mathematics, and that no certificate can contain. That is the step that connects the formalism to the world, forcing us to listen carefully to the structures themselves.

And it is at the connection to the world that things fail. When a bridge collapses, when an encryption system is broken, when an artificial superintelligence is misaligned, the question is never “was the certificate valid?” The certificate was valid. The question is: what was modeled that should not have been? What was optimized at the expense of what? No system can be the sole auditor of its own presuppositions. Someone standing outside it must be able to ask the right question: a human being carrying with her three thousand years of tradition, with the training and the judgment to find, in a chain of impeccable reasoning, the one link that was attached to nothing.

The physicist Carl Sagan called scientific understanding a candle in the dark [4], our protection against a world of technology we use but cannot evaluate, governed by claims we cannot test. We are now building that world without guardrails. We are rapidly approaching a condition where we will live surrounded by marvelous devices whose operation we do not comprehend, based on engineering principles we cannot audit, derived from formalisms we cannot interpret. When everything works, this is merely eerie. When something fails, it is the difference between a diagnosis and a mystery.

This is the service that mathematical culture provides: not merely theorems, but judgment. It is the trained capacity to assess mathematical claims, examine their assumptions, and explain what they do and do not establish; to distinguish the rigorous from the plausible; and to insist, stubbornly, that “it works” is not the same as “we understand why it works.”

VI. THE PATROL MUST CONTINUE

Pure mathematics looks, from the outside, like a spectacular waste of talent and resources: thousands of highly trained minds engaged in activities of zero practical consequence. Its products are unreadable to outsiders. Its practitioners spend years on problems whose solution will be celebrated by a handful of specialists and unrecognized by everyone else.

And this is precisely why it works.

The apparent “uselessness” of pure mathematics is the condition of its strength. It is what ensures that the habits of rigor, precision, honesty, patience, and the subordination of wishful thinking to proof are cultivated for their own sake, less governed by immediate commercial demands. A mathematician who proves a theorem about the distribution of prime numbers is not trying to break anyone’s encryption. She is trying to see the truth of arithmetic because her insights led her there.

The fact that her understanding might, decades later, be relevant to encryption is a consequence of the depth of her inquiry, not its purpose.

If you redirect this community toward “useful” problems, if you demand that mathematicians justify their work in terms of immediate applications, if you fund only research with identifiable economic impact, if you replace exploratory inquiry with targeted problem-solving, you do not get more useful mathematics. You get shallower mathematics. You get mathematics that is optimized for the next grant cycle rather than the next century. You get mathematics that has lost the very quality, its disinterested depth, that made it useful in the first place.

And you get a generation of students who have learned to solve problems, but not to think; who can operate a system, but not see its limitations; who can follow rules, but not imagine alternatives.

The patrol, the deliberate wandering, must continue, not because any individual circuit of the borders matters, but because the patrol itself is what keeps the borders safe.

VII. WHAT IS TO BE DONE

The world we live in changes through capital, policy, and law. It is institutions that house us, and we must pay attention to the realities facing the elected officials who oversee government funding, the university administrators tasked with balancing budgets, and above all the taxpaying public, who trust our standards. Elsewhere, I have proposed specific mechanisms for preserving mathematical capacity [5]; three of them form the core of what is needed now.

First, **independent offices of mathematical assurance**: standing public bodies, funded by governments rather than by AI companies, whose mathematicians audit the foundational logic and geometric architecture of machine-generated reasoning wherever it enters consequential systems, including infrastructure, cryptography, medicine, and finance. Mathematicians are not there to replace physicians or civil engineers, but to audit the structural mathematics upon which those specialists increasingly rely. The model is the air-safety investigator: independent, uncompromising, and answerable to the public.

Second, **comprehension residencies at mathematical institutes**: funded positions devoted to understanding major machine-produced proofs, reconstructing them, simplifying them, and extracting the ideas that a bare certificate need not make intelligible. Here the deliverable is not a publication, but an explanation.

Third, **reinforced graduate programs**: training that fiercely protects ongoing, year-long foundational courses in pure mathematics while sustaining a seminar culture at every level, from graduate student reading groups to invited colloquia and faculty-organized research seminars. The ambitious seminar list at the department entrance cannot exist without coursework and student inquiry beneath it. Integrating the interpretation and verification of machine reasoning into this

training does not compromise the tradition, but extends it, preparing students for a world in which the capacity to audit a mathematically sophisticated argument is scarcer, and therefore more valuable, than the capacity to generate one.

These proposals do not contradict the defense of “useless” exploratory mathematics; they depend on it. Auditing and comprehension cannot be taught in isolation as narrow procedural skills; they are byproducts of the deep, undirected wandering that pure mathematics sustains. The mathematician who can audit an artificial intelligence is able to do so precisely because her judgment was forged by contending with the abstract spirit of geometry. If we protect the exploratory core of the discipline, we preserve the rigorous discernment required for this work. The residencies and assurance offices serve as the bridge, ensuring that the culture of pure mathematics remains economically viable and practically integrated without sacrificing its disinterested depth.

We stand at the beginning of a profound transformation. The question is whether, when that transformation is complete, there will still be human beings who can look at a complex chain of mathematical reasoning, generated by any algorithmic system, and say, with the authority and conceptual clarity earned through years of practice: *this is what the result establishes, and what it does not, and here is why.*

If the answer is yes, the wandering continues; the borders hold, and our civilization keeps its sight.

If the answer is no, we will have all the theorems we could ever want, proved by machines of extraordinary power, verified by systems of impeccable precision. And no one will really understand any of them. And no one, when the system fails, will know how to ask the right question.

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Somewhere tomorrow morning a graduate student will stop in front of a seminar list and read a title she does not yet understand. Everything this essay defends begins in that moment.

References

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